University of Groningen

Standard errors as weights in multilateral price indices

Hill, Robert; Timmer, Marcel

Standard Errors as Weigths in Multilateral Price Indices

Research Memorandum GD-73

Robert Hill and Marcel Timmer

Groningen Growth and Development Centre
November 2004

Standard Errors as Weights in Multilateral Price Indexes

Robert J. Hill
School of Economics, University of New South Wales, Sydney 2052, Australia
E-Mail: r.hill@unsw.edu.au

Marcel P. Timmer
Faculty of Economics, University of Groningen, 9700 Av Groningen, The Netherlands.
Email: m.p.timmer@eco.rug.nl

September 22, 2004

Abstract: A number of multilateral methods for computing price indexes use bilateral comparisons as their basic building blocks. Some of these methods, such as the weighted-EKS and minimum-spanning-tree (MST) methods, give greater weight to those bilateral comparisons that are deemed more reliable (an adjustment that is particularly important for a heterogeneous set of countries). No consensus currently exists in the literature as to the best measure of reliability. Diewert (2002), in particular, proposes a number of reliability measures in an axiomatic setting. Existing measures (including all of Diewert's), however, fail to penalize bilateral comparisons when there is a small overlap in the products priced by each country. It is exactly in such situations that weighted methods are potentially most useful, but only if the reliability measure penalizes bilateral comparisons containing lots of gaps. Using a stochastic model, we show how the standard errors on bilateral price indexes provide a natural measure of reliability that automatically penalizes comparisons containing lots of gaps. Furthermore, we link these standard errors with the existing literature by showing that they are a generalization of one of Diewert's reliability measures. This finding provides an interesting new link between the axiomatic and stochastic approaches to index numbers. Also, these standard errors can be modified for use in consumer data sets below the basic-heading level (where no expenditure shares are available), a scenario of direct relevance to the latest round of the International Comparison Program (ICP) currently being undertaken at the World Bank. Finally, we apply our methodology to an international data set on agricultural production that contains a lot of gaps. Our results clearly demonstrate the appeal of weighted methods and the importance of adjusting the reliability measures for gaps in the data. Failure to do so may compromise weighted methods precisely in situations where they are most needed. (*JEL*. C43, E31, O47)

KEYWORDS: Weighted Multilateral Method; Stochastic Approach; Axiomatic Approach; Törnqvist Price Index; Dissimilarity Measure; EKS; Spanning Tree; Missing Data; International Comparisons Program (ICP)

We thank participants at the UC Davis Workshop on Estimating Production and Income Across Nations, April 13-14, 2004 (particularly Erwin Diewert) for helpful comments.

I. Introduction

Comparing price levels and living standards across countries is an issue of interest to national governments, firms and households, and international organizations such as the International Monetary Fund (IMF), World Bank and European Union (EU). Such comparisons can influence budget contributions to international organizations and aid flows. They are also relevant to the fields of development and international economics, and the literature on economic convergence.

A number of multilateral methods (i.e., methods that generate internally consistent results for three or more countries) have been proposed in the index-number literature for making such comparisons. A distinction can be drawn between methods that use bilateral comparisons as their basic building blocks, and those that do not. Methods in the former category include the EKS (see Eltetö and Köves, 1964, and Szulc, 1964), weighted-EKS (see Rao, 1999) and minimum-spanning-tree (MST) (see Hill, 1999) methods, while methods in the latter category include the Geary (1958)-Khamis (1972), Iklé (1972) and weighted Country-Product Dummy (WCDP) (see Rao, 1999 and Diewert, 2004a) methods.

One attraction of building a multilateral comparison up from bilateral comparisons is that it opens up the possibility of discriminating between bilateral comparisons on the basis of their reliability, and giving greater weight to those that are more reliable. For example, a comparison between France and Germany may be deemed more reliable than one between France and India, since there is a greater overlap in the products bought in France and Germany. This observation provides the underlying rational for both the weighted-EKS and MST methods. However, both methods require the reliability of bilateral comparisons to be quantified.

We survey the existing literature on reliability measures, particularly Paasche-Laspeyres spreads (see Hill, 1999) and Diewert's (2002) relative price dissimilarity measures . One deficiency of this literature is that none of the measures make adjustments for the number of common headings in each bilateral comparison. When there are few gaps in the matrices of prices and quantities over which the price indexes are constructed, this is of little consequence. However, in data sets containing many gaps, the standard reliability measures can generate highly misleading results. It is precisely for such data sets that weighted methods are potentially most useful, but only if the reliability measure penalizes bilateral comparisons containing lots of gaps. This is because, other things equal, a bilateral comparison made over a larger basket of products is likely to be more reliable than a comparison made over a smaller basket of products. Moreover, other things are not equal. The sheer fact that a pair of countries have very little overlap in the products produced (or consumed) indicates that these countries are very different and by implication hard to compare. These points are clearly illustrated in the empirical part of the paper.

The objective of this paper is to develop a measure of reliability that penalizes in a nonarbitrary way bilateral comparisons containing a large number of gaps. We depart from the existing literature, which is typically couched in an axiomatic setting, by approaching the problem from a stochastic perspective. It is shown how standard errors on the logarithms of Törnqvist price indexes, derived from a stochastic model, provide a natural measure of reliability which automatically penalizes bilateral comparisons containing lots of gaps. We argue, therefore, that using standard errors as reliability measures will improve the quality of the results generated by weighted-EKS and MST.

Our findings are also interesting in that they provide a bridge between the stochastic and axiomatic approaches to index number theory by showing that one of Diewert's measures (derived in an axiomatic setting) is a special case of our measure (derived in a stochastic setting).

We show as well how our standard errors can be modified for use in consumer data sets below the basic-heading level (where no expenditure shares are available). This illustration is of direct relevance to the latest round of the International Comparison Program (ICP) currently being undertaken at the World Bank, which is planning to use a variant on a method used by Eurostat (the statistical office of the European Union).

The Eurostat method requires countries to identify products for which they supply price data as representative or unrepresentative. These representative or unrepresentative identifiers can be used to construct proxy expenditure shares (each representative product is allocated an equal share and unrepresentative products are allocated a zero share), thus allowing our standard errors to be computed. This means that weighted binary-based multilateral methods can even be applied below basic heading level. The application of weighted methods below basic heading level may prove useful to the ICP where the groupings of countries can be quite diverse and gaps in the data are pervasive.

We conclude the paper with an empirical comparison of our reliability measure and its resulting weighted-EKS and MST price indexes with those obtained using other measures. The data set consists of agricultural producer prices and quantities for 181 agricultural products (crops) for the year 1995. The data set covering 103 countries was constructed by Rao, Ypma and van Ark (2003) from a FAOSTAT, agricultural and producer prices database. The interesting feature of the data set, from our perspective, is that it covers a large and diverse set of countries and contains a lot of gaps. The presence of many gaps in such a data set is inevitable. For example, it is not surprising that tropical foods and spices are not grown in Norway. If we want to find the most reliable bilateral comparisons in this data set, it is crucial that we take account of the amount of overlap of crops in each bilateral comparison. We show that failure to make such adjustments can lead to the selection of highly undesirable bilateral links, thus compromising weighted binary-based multilateral methods such as weighted-EKS and MST precisely in situations where they are most needed (i.e., in a comparison over a heterogeneous set of countries).

2. Bilateral and Multilateral Price Indexes

(i) Bilateral Price Indexes

The set of countries is indexed by $k = 1, \ldots, K$ and the set of commodity headings by $n = 1, \ldots, N_{jk}$. Here we allow for the possibility that the set of headings over which each bilateral comparison is made may not be identical. This is the reason for the jk subscript on N. The price and quantity data of heading n in country k are denoted, respectively, by p_{kn} and q_{kn}.

Let P_{jk} and Q_{jk} denote, respectively, bilateral price and quantity index comparisons between countries j and k. Four important bilateral index number formulae are defined below:[1]

$$\text{Laspeyres}: \quad P_{jk}^{L} = \frac{\sum_{n=1}^{N_{jk}} p_{kn} q_{jn}}{\sum_{n=1}^{N_{jk}} p_{jn} q_{jn}}, \qquad Q_{jk}^{L} = \frac{\sum_{n=1}^{N_{jk}} p_{jn} q_{kn}}{\sum_{n=1}^{N_{jk}} p_{jn} q_{jn}} \tag{1}$$

$$\text{Paasche}: \quad P_{jk}^{P} = \frac{\sum_{n=1}^{N_{jk}} p_{kn} q_{kn}}{\sum_{n=1}^{N_{jk}} p_{jn} q_{kn}}, \qquad Q_{jk}^{P} = \frac{\sum_{n=1}^{N_{jk}} p_{kn} q_{kn}}{\sum_{n=1}^{N_{jk}} p_{kn} q_{jn}} \tag{2}$$

$$\text{Fisher}: \quad P_{jk}^{F} = \sqrt{P_{jk}^{P} P_{jk}^{L}}, \qquad Q_{jk}^{F} = \sqrt{Q_{jk}^{P} Q_{jk}^{L}} \tag{3}$$

$$\text{Törnqvist}: \quad P_{jk}^{T} = \prod_{n=1}^{N_{jk}} \left(\frac{p_{kn}}{p_{jn}} \right)^{(s_{jn}+s_{kn})/2} \qquad Q_{jk}^{T} = \prod_{n=1}^{N_{jk}} \left(\frac{q_{kn}}{q_{jn}} \right)^{(s_{jn}+s_{kn})/2} \tag{4}$$

$$\text{where} \quad s_{jn} = \frac{p_{jn} q_{jn}}{\sum_{i=1}^{N_{jk}} p_{ji} q_{ji}}.$$

One weakness of these formulae is that they are not transitive. For example, in general, $P_{jk}^{F} P_{kl}^{F} \neq P_{jl}^{F}$.

(ii) Multilateral Price Indexes

A multilateral price index, by construction, is transitive. Multilateral price indexes for countries j and k are denoted here, respectively, by P_j and P_k. A bilateral comparison of prices made using a multilateral formula can be expressed as follows:

$$P_{jk} = \frac{P_k}{P_j}.$$

Transitivity is achieved by sacrificing independence of irrelevant alternatives (see van Veelen, 2002). That is, the ratio P_k/P_j, in general, will depend not only on the price and quantity vectors of countries j and k, but also on the price and quantity vectors

[1]The history of these formulae is discussed in Diewert (2001).

of some or all of the other countries in the comparison. A large number of multilateral formulae have been proposed in the index number literature (see for example Balk, 1996, Hill, 1997, and Diewert, 1999, for surveys of this literature).

3. Binary-Based Multilateral Methods

(i) Star Methods

Graph Theory provides a useful framework for analyzing the underlying structure of multilateral price indexes. A graph consists of a collection of vertices linked by edges. In the context of spatial comparisons, each vertex represents one of the countries in the comparison, while each edge represents a bilateral comparison between a pair of countries. Two particularly important graphs, depicted in Figure 1 for the case of 5 vertices, are the *star* and *complete* graphs.

Insert Figure 1 Here

Perhaps the simplest multilateral method is the star method, which uses the star graph. The star method places one country, denoted here by b, at the center of the star. The multilateral price index for country k is then defined as $P_k = P_{bk}$, where P_{bk} is a bilateral price index such as Fisher or Törnqvist. This means that a comparison between countries j and k is made by linking together bilateral comparisons between countries j and b and countries b and k.

However, the fact that the bilateral formulae are not transitive implies that the multilateral price indexes depend on which country is placed at the center of the star. For example, suppose country b is replaced with country d at the center of the star. In general,

$$\frac{P_{bk}}{P_{bj}} \neq \frac{P_{dk}}{P_{dj}}.$$

This is the main weakness of the star method. In most applications, it is not clear which country should be placed at the center of the star.[2]

[2] Methods such as Geary (1958)-Khamis (1972) and Iklé (1972) solve this problem by putting an artificial average country at the center of the star.

(ii) The EKS Method

The EKS method, named after Eltetö and Köves (1964) and Szulc (1964) but first proposed by Gini (1931), also uses the star graph. It manages to treat all countries symmetrically by generating K series of multilateral price indexes, each with a different country at the center of the star. These K series of results are then averaged. The EKS method usually uses the Fisher index to make the bilateral comparisons. The Törnqvist version of EKS is often referred to as the CCD method (see Caves, Christensen and Diewert, 1982).[3] The EKS formula transitivizes the bilateral price indexes as follows:

$$P_k = \prod_{j=1}^{K} \left[(P_{jk})^{1/K} \right]. \tag{5}$$

EKS is the preferred multilateral method of Eurostat and the OECD.

The EKS method can also be described using a complete graph (see Figure 1) since it uses bilateral comparisons between all possible pairs of countries. A total of $K(K-1)/2$ distinct bilateral comparisons are defined on a set of K vertices. Inevitably, some of these bilateral comparisons are likely to be more reliable than others. This observation provides the rationale for the weighted-EKS method discussed below.

(iii) The Weighted-EKS Method

The weighted-EKS method, proposed by Rao (1999) and discussed in greater detail in Rao (2001) and Rao and Timmer (2003), allows each bilateral comparison to be given different weight in the multilateral comparison.

The EKS price indexes P_j and P_k can be obtained as the solution to the following minimization problem:

$$\min_{\ln P_j, \ln P_k} \quad \sum_{j=1}^{K} \sum_{k=1}^{K} (\ln P_k - \ln P_j - \ln P_{jk}^F)^2,$$

where the normalization $P_1 = 1$ is imposed. The solutions, $\ln \hat{P}_j, \ln \hat{P}_k$ are the ordinary

[3] One attractive feature of the CCD method is that it can also be represented as a star method with an artificial country at the center of the star.

least squares estimators of $\ln P_j, \ln P_k$ in the model:

$$\ln P_{jk}^F = \ln P_k - \ln P_j + \epsilon_{jk}, \tag{6}$$

with $E(\epsilon_{jk}) = 0$ and $\text{var}(\epsilon_{jk}) = \sigma^2$.

Rao's weighted-EKS method assumes instead that the errors are heteroscedastic, i.e.,

$$\text{var}(\epsilon_{jk}) = \sigma^2/w_{jk}, \text{ for } j \neq k, \text{ and } \text{var}(\epsilon_{jj}) = 0. \tag{7}$$

The weights, w_{jk}, measure the reliability of the comparison between countries j and k. The specification of the weights, w_{jk}, is discussed later. For now, we treat them as given. The complete matrix of weights is denoted here by W.[4]

$$W = \begin{pmatrix} 0 & w_{12} & \cdots & w_{1K} \\ w_{21} & 0 & \cdots & w_{2K} \\ \vdots & \vdots & & \vdots \\ w_{K1} & w_{K2} & \cdots & 0 \end{pmatrix}$$

The weighted-EKS price indexes, P_k, are obtained as follows:

$$\begin{pmatrix} \ln P_2 \\ \ln P_3 \\ \vdots \\ \ln P_K \end{pmatrix} = \begin{pmatrix} \sum_{j\neq 2}^K w_{2j} & -w_{23} & \cdots & -w_{2K} \\ -w_{32} & \sum_{j\neq 3}^K w_{3j} & \cdots & -w_{3K} \\ \vdots & \vdots & & \vdots \\ -w_{K2} & -w_{K3} & \cdots & \sum_{j\neq K}^K w_{Kj} \end{pmatrix}^{-1} \begin{pmatrix} -\sum_{j\neq 2}^K w_{2j} \ln P_{2j}^F \\ -\sum_{j\neq 3}^K w_{3j} \ln P_{3j}^F \\ \vdots \\ -\sum_{j\neq K}^K w_{Kj} \ln P_{Kj}^F \end{pmatrix}.$$

The price index for country 1, P_1, is normalized to 1. In the case where $w_{jk} = w$ for all j, k, the weighted-EKS method reduces to the standard EKS formula in (5).

(iv) The Minimum-Spanning-Tree (MST) Method

A multilateral comparison between K countries can be made by simply chaining together $K - 1$ bilateral comparisons (edges), as long as the underlying graph is a

[4]Presumably the matrix W must be symmetric. Also, if a particular bilateral comparison is assigned a weight of zero, in equation (7) this comparison should be interpreted as having an infinite variance. Hence it plays no part in the determination of the weighted-EKS price indexes.

spanning tree (see Hill, 1999). A spanning tree is a connected graph that does not contain any cycles. In other words, any pair of vertices in the graph are connected by one and only one path of edges. The reason why there must be no cycles in the graph is to ensure that the multilateral price indexes are transitive and hence internally consistent. A total of K^{K-2} different spanning trees are defined on a set of K vertices. Three examples of spanning trees defined on the set of 9 vertices are shown in Figure 2.[5]

Insert Figure 2 Here

The resulting set of multilateral price indexes depends both on the choice of formula used for making the bilateral comparisons and on the choice of spanning tree. The bilateral comparisons should be made using a superlative formula such as Fisher or Törnqvist.[6] Since superlative formulae satisfy the country reversal test (i.e., $P_{jk} = 1/P_{kj}$), there is no need for directional arrows on the edges in the spanning tree to identify the base country in each bilateral comparison.

The choice of spanning tree is more problematic. A criterion is needed for deciding which edges (bilateral comparisons) to include and which to exclude. As with the weighted-EKS method this requires a weight to be placed on each bilateral comparison. Again, the specification of weights is deferred until later.[7] The minimum-spanning tree can be derived using Kruskal's algorithm. Kruskal's algorithm proceeds by selecting sequentially the bilateral comparisons (edges) with the smallest weights subject to the constraint that adding that edge to the graph does not create a cycle. The algorithm terminates once it is no longer possible to select any more edges without creating a cycle. It turns out that the resulting spanning tree has the minimum sum of weights.

[5]The star graph in Figure 1 is another example of a spanning tree.

[6]See Diewert (1976) for a definition and discussion of the properties of superlative indexes.

[7]In the context of the minimum-spanning-tree method as described in Hill (1999), and unlike Rao's weighted-EKS method, the greater the reliability of a bilateral comparison the smaller its weight. The MST method can easily be reformulated to use the same weights as the weighted-EKS method. In this case the only difference is that what is required now is the maximum spanning tree rather than the minimum spanning tree.

This is a well-known theorem in the Graph Theory literature (see for example Wilson, 1985).[8]

4. A Survey of Existing Measures of Reliability for Bilateral Comparisons

Both the weighted-EKS and minimum-spanning-tree (MST) methods require the construction of a matrix of weights. Ideally, we should use whichever bilateral comparisons are most reliable. One important difference between the two methods is that MST price indexes depend only on the ordinal ranking of the reliability measures. Hence MST price indexes, unlike weighted-EKS price indexes, are unaffected by monotonic transformations of the reliability measure. For example, the logarithmic transformation of the PLS measure defined below in (8) matters for the weighted-EKS method, but is of no consequence for the MST method.

The literature on measures of reliability for bilateral comparisons has for the most part been developed in an axiomatic setting. When discussing the sensitivity of a bilateral comparison to the choice of index number formula it is useful first to consider the limiting cases where all formulae give the same answer. The data are consistent with the conditions for Hicks's aggregation theorem if $p_{kn} = \lambda p_{jn}$ for $n = 1, \ldots, N_{jk}$, where λ denotes a positive scalar. In this case, all price index formulae reduce to λ. The data are consistent with the conditions for Leontief's aggregation theorem if $q_{kn} = \mu q_{jn}$ for $n = 1, \ldots, N_{jk}$, where μ again is a positive scalar. In this case, all quantity index formulae reduce to μ. It follows, therefore, that all price index formulae should reduce to $(\sum_{n=1}^{N_{jk}} p_{kn} q_{kn})/(\mu \sum_{n=1}^{N_{jk}} p_{jn} q_{jn})$, since price indexes can be obtained implicitly from quantity indexes.

One measure of sensitivity that has received attention in the index number literature is the Paasche-Laspeyres spread (PLS). This is usually defined as some function of the

[8] In some sense the MST method can be thought of as a special case of the weighted-EKS method for which $2(K-1)$ of the elements of the W matrix are set equal to one, and all other elements equal zero.

ratio of a Paasche price index to a Laspeyres price index.[9] For example, Hill (1999) defines it as follows:

$$PLS_{jk} = \ln\left[\frac{\text{Max}(P_{jk}^P, P_{jk}^L)}{\text{Min}(P_{jk}^P, P_{jk}^L)}\right]. \tag{8}$$

The PLS has the attractive property that it equals zero if the data satisfy the conditions for either Hicks or Leontief aggregation. When either condition is satisfied there is no index number problem, since all formulae should give the same answer. This suggests that we can have a high degree of confidence in the results of a bilateral comparison with a small PLS, since the underlying data are broadly consistent with either Hicks or Leontief aggregation. However, the link between the PLS and Hicks or Leontief aggregation is not exact, since the PLS can equal zero even when the conditions for Hicks and Leontief aggregation are both violated.

For this reason, Diewert (2002) advocates separate measures of sensitivity for price and quantity indexes, which he refers to as relative dissimilarity measures. He considers the axiomatic properties of a number of alternative measures. His relative dissimilarity measures for prices (quantities) all share the characteristic that they equal zero if and only if the data satisfy the conditions for Hicks (Leontief) aggregation. One of his preferred measures is

$$S_{jk}^P \equiv \sum_{n=1}^{N_{jk}} \left\{ \left(\frac{s_{jn}+s_{kn}}{2}\right) \left[\ln\left(\frac{1}{P_{jk}^T}\frac{p_{kn}}{p_{jn}}\right)\right]^2 \right\}, \tag{9}$$

$$S_{jk}^Q \equiv \sum_{n=1}^{N_{jk}} \left\{ \left(\frac{s_{jn}+s_{kn}}{2}\right) \left[\ln\left(\frac{1}{Q_{jk}^T}\frac{q_{kn}}{q_{jn}}\right)\right]^2 \right\}, \tag{10}$$

where S_{jk}^P and S_{jk}^Q denote the price and quantity dissimilarity measures, and P_{jk}^T and Q_{jk}^T denote Törnqvist price and quantity indexes as defined in (4).

If desired, S_{jk}^P and S_{jk}^Q can be combined as follows:

$$S_{jk} \equiv \min(S_{jk}^P, S_{jk}^Q). \tag{11}$$

[9] The ratio of Paasche to Laspeyres is the same for price and quantity indexes.

This measure, a variant on which is used by Hill (2004), equals zero if and only if the data are consistent with either Hicks or Leontief aggregation.[10,11]

All the reliability measures discussed above, however, share one fundamental weakness. They all assume that there are no gaps in the data. As soon as there are gaps, this means that some bilateral comparisons will be made over larger baskets of products than others. Other things equal we should prefer bilateral comparisons made over larger baskets. Furthermore, the sheer fact that a pair of countries have very little overlap in the products produced (or consumed) indicates that these countries are very different and by implication hard to compare. Therefore, ideally a measure of reliability should penalize bilateral comparisons where the overlap of products is small. At first glance it seems that any such adjustment must be arbitrary. However, by approaching the problem from a stochastic perspective, in the next section we derive a reliability measure that naturally makes such an adjustment. In addition, although we approach the problem from a very different perspective than Diewert (2002) who uses an axiomatic approach, it emerges that our reliability measure is a generalization of one of his measures.

5. A Stochastic Approach to the Measurement of Reliability

In this section we show how if the same problem of measuring the reliability of bilateral comparisons is approached from the stochastic perspective, we obtain standard errors on the logarithms of Törnqvist price indexes that can serve as measures of the reliability of bilateral comparisons. Furthermore, we link these standard errors with the existing literature by showing that they are in fact a generalization of one of Diewert's

[10]For most data sets it will be the case that $S^P_{jk} < S^Q_{jk}$, and hence S_{jk} will simplify to S^P_{jk}. This empirical regularity was noted by Allen and Diewert (1981).

[11]It is worth noting that when $N_{jk} = 1$ (i.e., the comparison is made over only one heading) $PLS_{jk} = S^P_{jk} = S^Q_{jk} = S_{jk} = 0$. For the measure to be meaningful, therefore, we must restrict attention to cases where $N_{jk} \geq 2$.

relative price dissimilarity measures. The main difference is that the standard errors contain an additional term that penalizes bilateral comparisons where the overlap of products is small. This finding provides an interesting new link between the axiomatic and stochastic approaches to index numbers.

Our stochastic model builds on the work of Clements and Izan (1981), Cuthbert and Cuthbert (1989) and Selvanathan and Rao (1994). These authors show how the stochastic approach can be used to derive standard errors on the logarithms of Paasche, Laspeyres and Törnqvist indexes, as functions of the number of observed price headings (see also Diewert, 1995). Although they do not draw attention to this issue (which is not surprising since these papers predate Diewert, 2002), it turns out that the standard errors on the logarithms of the Törnqvist price indexes derived by Clements and Izan (1981) and Selvanathan and Rao (1994) differ from Diewert's dissimilarity measure S_{jk}^P only in that they make a adjustment by dividing S_{jk}^P by $N_{jk}-1$. Hence these standard errors do make a simple adjustment for gaps in the data. However, the adjustment is not entirely satisfactory since it does not take account of the value share of each product. Our model differs slightly from theirs and in the process generates a more satisfactory method of adjustment that explicitly factors in value shares. The approach of Cuthbert and Cuthbert (1989) is slightly different again, in that it focuses on comparisons below the basic heading level. Cuthbert and Cuthbert's contribution is discussed later in the paper.

It is useful to begin with a discussion of Clements and Izan's stochastic model of the Törnqvist price index. They assume that the price relatives can be modelled as follows:

$$\ln\left(\frac{p_{kn}}{p_{jn}}\right) = \alpha_{jk} + \varepsilon_{jk,n}. \tag{12}$$

The term α_{jk} in (12) represents the systematic part of the difference in the purchasing power of the currencies of countries j and k, while $\varepsilon_{jk,n}$ denotes the random element. Clements and Izan assume that the errors are independently distributed as follows:

$$E(\varepsilon_{jk,n}) = 0, \quad \mathrm{Var}(\varepsilon_{jk,n}) = \frac{\sigma_{jk}^2}{N_{jk}w_{jk,n}}, \tag{13}$$

where $w_{jk,n} > 0$ denotes a nonrandom weight attached to heading i, such that $\sum_{n=1}^{N_{jk}} w_{jk,n}$ $= 1$. It is assumed that $N_{jk} \geq 2$. Also, here we have added N_{jk} to the denominator of the variance term so that the average variance of a heading is independent of the number of headings. This adjustment is necessary to make the results consistent with a different version of the model discussed later.

The important point about Clement and Izan's specification is that it presumes that the price relatives of headings with larger weights (expenditure shares) approximate more closely the underlying price index. This is unlikely to be the case for most data sets. For example, the largest expenditure shares often belong to hard-to-measure categories such as health and education.

Continuing for the moment with our slightly modified version of the Clements and Izan model, it follows from (12) and (13) that the generalized-least-squares estimator of α_{jk} is

$$\hat{\alpha}_{jk} = \sum_{n=1}^{N_{jk}} \left[w_{jk,n} \ln \left(\frac{p_{kn}}{p_{jn}} \right) \right]. \tag{14}$$

When $w_{jk,n} = (s_{jn} + s_{kn})/2$, $\hat{\alpha}_{jk}$ reduces to the logarithm of the Törnqvist price index. It follows from (13) that

$$\text{Var}(\hat{\alpha}_{jk}) = \sum_{n=1}^{N_{jk}} \left[w_{jk,n}^2 \left(\frac{\sigma_{jk}^2}{N_{jk} w_{jk,n}} \right) \right] = \frac{\sigma_{jk}^2}{N_{jk}}. \tag{15}$$

An unbiased estimator of σ_{jk}^2 is

$$\hat{\sigma}_{jk}^2 = \left(\frac{N_{jk}}{N_{jk} - 1} \right) \sum_{n=1}^{N_{jk}} \left\{ w_{jk,n} \left[\ln \left(\frac{p_{kn}}{p_{jn}} \right) - \hat{\alpha}_{jk} \right]^2 \right\}. \tag{16}$$

Returning to (13), suppose now we modify the underlying assumptions as follows:

$$E(\varepsilon_{jk,n}) = 0, \quad \text{Var}(\varepsilon_{jk,n}) = (\sigma_{jk})^2, \tag{17}$$

so that the variances of the errors are independent of the weights. We believe this assumption is more realistic.

Our second departure from the approach used by Clements and Izan is that instead of generalized least squares in a heteroscedastic model, we use weighted least squares in a homoscedastic model to estimate α_{jk}, with the weight on observation n equal to $w_{jk,n}$:

$$\min_{\alpha_{jk}} \sum_{n=1}^{N_{jk}} \left\{ w_{jk,n} \left[\ln\left(\frac{p_{kn}}{p_{jn}}\right) - \alpha_{jk} \right]^2 \right\}. \tag{18}$$

Solving (18), we obtain the following estimator of α_{jk}:

$$\tilde{\alpha}_{jk} = \sum_{n=1}^{N_{jk}} \left[w_{jk,n} \ln\left(\frac{p_{kn}}{p_{jn}}\right) \right]. \tag{19}$$

Our approach here is somewhat analogous to that used in the weighted Country-Product-Dummy (WCPD) method (see Rao, 2001). Using weighted least squares makes sense in our context since the price relatives for headings with larger weights (value shares) are more important. For example, in a consumer context, suppose product 1 accounts for 10 percent and 30 percent, respectively, of total expenditure in countries j and k, while product 2 accounts for only 0.01 percent and 0.03 percent, respectively. Then it is clear that the price relative for product 1 should exert greater influence on $\tilde{\alpha}_{jk}$.

A comparison of (14) and (19) reveals that $\hat{\alpha}_{jk}$ and $\tilde{\alpha}_{jk}$ are the same. However, the variances of $\hat{\alpha}_{jk}$ and $\tilde{\alpha}_{jk}$ are not the same. The variance of $\tilde{\alpha}_{jk}$ in (20) only reduces to σ_{jk}^2/N_{jk} if $w_{jk,n} = 1/N_{jk}$ for $n = 1, \ldots, N_{jk}$.

$$\mathrm{Var}(\tilde{\alpha}_{jk}) = \sigma_{jk}^2 \sum_{n=1}^{N_{jk}} w_{jk,n}^2 \tag{20}$$

An unbiased estimator of σ_{jk}^2 for this model is

$$\tilde{\sigma}_{jk}^2 = \left(\frac{1}{1 - \sum_{n=1}^{N_{jk}} w_{jk,n}^2} \right) \sum_{n=1}^{N_{jk}} \left\{ w_{jk,n} \left[\ln\left(\frac{p_{kn}}{p_{jn}}\right) - \tilde{\alpha}_{jk} \right]^2 \right\}. \tag{21}$$

This result is derived as follows:

$$E\left[\sum_{n=1}^{N_{jk}} w_{jk,n}(x_{jk,n} - \tilde{\alpha}_{jk})^2 \right] = \sum_{n=1}^{N_{jk}} [w_{jk,n} E(x_{jk,n}^2)] - [E(\tilde{\alpha}_{jk})]^2$$

$$= \sigma_{jk}^2 + \alpha_{jk}^2 - \sigma_{jk}^2 \sum_{n=1}^{N_{jk}} w_{jk,n}^2 - \alpha_{jk}^2$$

$$= \sigma_{jk}^2 \left(1 - \sum_{n=1}^{N_{jk}} w_{jk,n}^2 \right),$$

where $x_{jk,n} = \ln(p_{kn}/p_{jn})$.

Equations (16) and (21) are equivalent when $w_{jk,n} = 1/N_{jk}$ for $n = 1, \ldots, N_{jk}$. Setting the weights $w_{jk,n} = (s_{jn} + s_{kn})/2$ in (16) and (21) we obtain that

$$\hat{\sigma}_{jk}^2 = \left(\frac{N_{jk}}{N_{jk} - 1} \right) S_{jk}^P, \tag{22}$$

$$\tilde{\sigma}_{jk}^2 = \left(\frac{1}{1 - \sum_{n=1}^{N_{jk}} [(s_{jn} + s_{kn})/2]^2} \right) S_{jk}^P. \tag{23}$$

That is, the estimated variance is a function of Diewert's relative price dissimilarity measure, S_{jk}^P. Replacing σ_{jk} with $\hat{\sigma}_{jk}$ in (15), we obtain that

$$\overline{\text{Var}}(\hat{\alpha}_{jk}) = \left(\frac{1}{N_{jk} - 1} \right) S_{jk}^P, \tag{24}$$

where $\overline{\text{Var}}(\hat{\alpha}_{jk})$ is an unbiased estimator of $\text{Var}(\hat{\alpha}_{jk})$. Similarly, replacing σ_{jk} with $\tilde{\sigma}_{jk}$ in (20), we obtain that

$$\overline{\text{Var}}(\tilde{\alpha}_{jk}) = \frac{\{\sum_{n=1}^{N_{jk}} [(s_{jn} + s_{kn})/2]^2\}}{\{1 - \sum_{n=1}^{N_{jk}} [(s_{jn} + s_{kn})/2]^2\}} S_{jk}^P, \tag{25}$$

where $\overline{\text{Var}}(\tilde{\alpha}_{jk})$ is an unbiased estimator of $\text{Var}(\tilde{\alpha}_{jk})$. Equation (25) provides a new measure of the variance of the logarithm of a Törnqvist price index.

It is important to consider the impact of N_{jk} on $\overline{\text{Var}}(\hat{\alpha}_{jk})$ and $\overline{\text{Var}}(\tilde{\alpha}_{jk})$. It is not clear in general, how an increase in N_{jk} (for $N_{jk} \geq 2$) will affect S_{jk}^P. However, an increase in N_{jk} must reduce $1/(N_{jk} - 1)$, and should reduce $\{\sum_{n=1}^{N_{jk}} [(s_{jn} + s_{kn})/2]^2\} / \{1 - \sum_{n=1}^{N_{jk}} [(s_{jn} + s_{kn})/2]^2\}$. By construction, s_{jn} and s_{kn} are calculated only over the products (headings) supplied by both countries j and k, (i.e., $\sum_{n=1}^{N_{jk}} s_{jn} = \sum_{n=1}^{N_{jk}} s_{kn} = 1$). Hence as N_{jk} rises, each of the s_{jn} and s_{kn} terms, on average, should fall which in turn should increase the numerator and reduce the denominator of (25) thus causing $\overline{\text{Var}}(\tilde{\alpha}_{jk})$ to rise. The empirical analysis later in the paper bears out these claims. Although $\overline{\text{Var}}(\hat{\alpha}_{jk})$ clearly penalizes (relative to S_{jk}^P) bilateral comparisons with smaller overlaps, the adjustment takes no account of the relative importance in terms of value shares of each product. It is for this reason that we believe $\overline{\text{Var}}(\tilde{\alpha}_{jk})$ is a superior measure of reliability.

Quantity versions of the stochastic models in (13) and (17) could also be developed. However, in this context it is more natural to treat the prices as stochastic and the quantities as responding to prices.

6. Choosing the Weights

(i) Above Basic-Heading Level

It is useful to draw a distinction between applications of price index methods above and below basic-heading level. Above basic-heading level expenditure shares are available, while below basic-heading level they are not. This distinction is mainly relevant to consumer data sets such as those used by the International Comparisons Program (ICP), the OECD and Eurostat which rely on consumer expenditure surveys to obtain value shares.

Focusing first on comparisons above basic-heading level (the usual case), if the Fisher index is replaced by Törnqvist in (6), and assuming heteroscedastic errors as in (7), we obtain the following model:[12]

$$\ln P_{jk}^{T} = \ln P_k - \ln P_j + \epsilon_{jk}, \quad \text{where} \;\; E(\epsilon_{jk}) = 0, \; \text{Var}(\epsilon_{jk}) = \frac{\sigma^2}{w_{jk}}. \tag{26}$$

It follows from (26) that

$$\text{Var}(\ln P_{jk}^{T}) = \text{Var}(\epsilon_{jk}) = \frac{\sigma^2}{w_{jk}}.$$

Finally, using equation (25) we obtain the following weights for the weighted-EKS and

[12]It is important not to confuse σ^2 with σ_{jk}^2. The former is the variance of the logarithmic deviations of Törnqvist price indexes from CCD price indexes in a multilateral comparison, while the latter is the variance of the logarithms of the price relatives in a bilateral comparison between countries j and k.

MST methods:[13]

$$w_{jk} = \frac{\sigma^2}{\overline{\mathrm{Var}}(\ln P^T_{jk})} = \frac{\{1 - \sum_{n=1}^{N_{jk}}[(s_{jn}+s_{kn})/2]^2\}}{\{\sum_{n=1}^{N_{jk}}[(s_{jn}+s_{kn})/2]^2\}}\frac{\sigma^2}{S^P_{jk}}. \tag{27}$$

The value of σ^2 in (27) has no effect on the resulting multilateral price indexes, and hence can be set equal to one.

(ii) Below Basic-Heading Level

Below basic heading level (i.e., at a level of detail lower than that provided in consumer expenditure surveys) expenditure shares are not available. This distinction between above and below basic heading level often arises in data sets covering consumer expenditure. The ICP, the OECD and Eurostat all have to construct price indexes at both levels. The latest round of the ICP, funded mainly by the World Bank, is currently reviewing alternative methodologies for making comparisons below basic heading level (see the ICP Handbook, 2004, and Diewert, 2004a).

A distinction can be drawn between cases where some products are identified as representative for a particular country (an approach pioneered by Eurostat) and cases where no such distinction is made. Considering first the latter scenario, it follows that $w_{jk,n} = 1/N_{jk}$ for all n. Therefore, the Törnqvist index in (4) reduces to the Jevons index (see Diewert, 2004b) defined below:

$$\text{Jevons}: \quad P^J_{jk} = \prod_{n=1}^{N_{jk}}\left[\left(\frac{p_{kn}}{p_{jn}}\right)^{1/N_{jk}}\right]. \tag{28}$$

Similarly, Diewert's price dissimilarity measure defined in (9) reduces to

$$S^P_{jk} = \frac{1}{N_{jk}}\sum_{n=1}^{N_{jk}}\left\{\left[\ln\left(\frac{1}{P^J_{jk}}\frac{p_{kn}}{p_{jn}}\right)\right]^2\right\}. \tag{29}$$

Equation (27) now reduces to

$$w_{jk} = \frac{\sigma^2(N_{jk}-1)}{S^P_{jk}},$$

[13]With the weights defined in this way, the MST method uses the maximum-spanning tree. Alternatively, if the minimum-spanning tree is used, the weights can be set simply equal to the variances in equation (25).

where S_{jk} is defined as in (29).

Suppose now that each country identifies some products as representative. This is the approach used by Eurostat and the ICP to construct price indexes below the basic heading level.[14] Now let $n = 1, \ldots, N_{jk}$ index the products that are representative in either country j or k and are priced in both j and k. Also, let N_{jk}^R denote the number of products that are representative in country j and priced in k, while N_{kj}^R denotes the number of products representative in country k that are priced in j.

The weights on each product can now be defined as follows: $w_{jk,n} = (s_{j,n}^* + s_{k,n}^*)/2$, where $s_{j,n}^* = 1/N_{jk}^R$ if product n is representative in country j, and $s_{j,n}^* = 0$ otherwise. Similarly, $s_{k,n}^* = 1/N_{kj}^R$ if product n is representative in country k, and $s_{k,n}^* = 0$ otherwise.[15] Using this approach, quasi-Törnqvist indexes can be computed with s_{jn} and s_{kn} replaced by s_{jn}^* and s_{kn}^* respectively in equation (4). Using these quasi expenditure shares it is likewise possible to compute S_{jk}^P and $\overline{\text{Var}}(\hat{\alpha}_{jk})$, and hence the methodology becomes analogous to the above basic heading case.

It must be remembered that in a multilateral comparison, this process is repeated for each heading (expenditure class) of which there may be around 200. Therefore, although the standard procedure should work in most cases, there will almost certainly be a few problematic headings where either N_{jk}^R or N_{kj}^R or both will equal zero in one or more of the bilateral comparisons. Clearly when this happens it is not possible to compute P_{jk}^T, S_{jk}^P and $\overline{\text{Var}}(\hat{\alpha}_{jk})$, since the weights $w_{jk,n}$ are not defined. For the purposes of the weighted-EKS and MST methods, in such cases $\overline{\text{Var}}(\hat{\alpha}_{jk})$ should be set to infinity. This ensures that this bilateral comparison is completely ignored by both methods. If, for a particular heading, this situation arises for so many bilateral comparisons that it is not possible to construct a spanning tree from the remaining comparisons (where N_{jk}^R and N_{kj}^R are both positive), then quasi-Törnqvist must be replaced by either quasi-

[14] See the ICP Handbook (2004) for further details on the OECD/Eurostat approach to comparisons at the elementary level.

[15] This means that any data available on a product that is not representative for either country is ignored, even if both countries price this product.

geometric Laspeyres or Paasche in the S_{jk}^P formulae. If both N_{jk}^R and N_{kj}^R equal zero, then use must be made of any unrepresentative price observations that are available for both countries.

The stochastic properties of the quasi-Törnqvist index have been considered previously by Cuthbert and Cuthbert (1989), although confusingly they refer to this index as a Fisher index (this terminology can be traced back to Eurostat). Cuthbert and Cuthbert derive an expression for the variance of the quasi-Törnqvist index which corresponds to equation (20), with the weights defined as above (see also Rao, 2001). Cuthbert and Cuthbert, however, do not derive an estimator for σ_{jk}^2 for their stochastic model and hence as it stands the variances cannot actually be computed (up to a scalar of proportionality) unless it is assumed that σ_{jk} is the same across all bilateral comparisons.

7. Empirical Application to an Agricultural Data Set

The data set consists of agricultural producer prices and quantities for 181 agricultural products (mainly crops) for the year 1995. Since quantities (and hence value shares) are available for each product, as is usually the case in producer data sets, the distinction between above and below basic heading level is not relevant. The weighting formula in (27) can be used directly. The data set covers 103 countries. It was constructed by Rao, Ypma and van Ark (2003) from a FAOSTAT agricultural and producer prices database. We have modified slightly Rao et al.'s original data set which contained 111 countries, by removing 8 countries. Singapore and Chad were removed due to a very limited number of agricultural products as were 6 Central Asian countries for which the price data were not original but appeared to be imputed from the Ukraine. (See Rao, Ypma and van Ark, 2003, for a full description of the data set.)

The interesting feature of the data set, from our perspective, is that it contains a lot of gaps. This is inevitable in a data set that covers most of the crops grown in the world. For example, it is not surprising that tropical foods and spices are not grown in Norway. If we want to find the most reliable bilateral comparisons in this data set, it is crucial that we take account of the amount of overlap of crops in each bilateral comparison.

The bilateral comparisons selected by the MST method are particularly illuminating. A total of 102 bilateral comparisons are selected (by construction one less than the number of countries in the comparison). These are listed in Table 1 in descending order for Diewert's measure, $1/S_{jk}^P$, and our measure, $1/\overline{\text{Var}}(\tilde{\alpha}_{jk})$. It is immediately apparent that many of the links for $1/S_{jk}^P$ are surprising. Some particularly eye-catching links that were among the first 25 selected (i.e., the best bilateral links in the MST) are: Norway-Niger (2), Norway-Malaysia (2), Norway-Indonesia (3), Malaysia-Canada (4), Mali-Georgia (4), Ghana-Canada (2). The size of the product overlap, N_{jk}, is provided in brackets after each link. The fact that these bilateral comparisons all have a low S_{jk}^P hardly implies that they are particularly reliable comparisons that should be used to construct a minimum-spanning tree or given high weights in the weighted-EKS method. Only two of these six links are selected by our measure, $1/\overline{\text{Var}}(\tilde{\alpha}_{jk})$, and in both cases they are further down the list. The MST links obtained using our measure are intuitively more plausible. Also, the average value of N_{jk} is 27.6 as opposed to 14.2 for Diewert's $1/S_{jk}^P$ measure. The difference is even more dramatic for the best 25 links. The average N_{jk} for this subset of links is 35.0 for our measure compared with 11.1 for $1/S_{jk}^P$.

Insert Table 1 Here

The problem with using weights based on S_{jk}^P is that when N_{jk} is small, it will contain a lot of noise. This means that in such cases there is a chance that S_{jk}^P could be small even though almost by definition countries j and k must be quite different. Given that the MST method selects the bilateral comparisons with the smallest dissimilarity measures, it will therefore pick up quite a few of these observations. However, in such cases it does not follow that these pairs of countries face similar relative prices for their agricultural products. In fact, when N_{jk} is small, the situation is quite the reverse since it implies that the mix of products they produce is very different, and hence it would be highly misleading to conclude that they face similar relative prices.

It is also important to compare the price indexes generated by the reliability measures $1/S_{jk}^P$ and $1/\overline{\mathrm{Var}}(\tilde{\alpha}_{jk})$ for both the weighted-EKS and MST methods. These results along with EKS-Törnqvist (i.e., CCD) price indexes are presented in Table 2 with the USA as the base country. Particularly striking are the results for Cameroon. The price index differs by a factor of 9 depending on which method is used!

Insert Table 2 Here

In the context of this paper, we are particularly interested in assessing the impact of the choice between using either $1/S_{jk}^P$ or $1/\overline{\mathrm{Var}}(\tilde{\alpha}_{jk})$ as weights on the weighted-EKS and MST price indexes. The dissimilarity (L_{jg}^b) of the results across a pair of methods (f and g) for a given base country (b) is measured here as follows:

$$L_{fg}^b = \frac{100}{K} \sum_{k=1}^{K} \left[\frac{\mathrm{Max}(P_k^f/P_b^f, P_k^g/P_b^g)}{\mathrm{Min}(P_k^f/P_b^f, P_k^g/P_b^g)} - 1 \right],$$

where P_k^f/P_b^f denotes the price index for country k obtained using multilateral method f with country b as the base. L_{fg}^b can be interpreted as measuring the average percentage difference between the price indexes generated by methods f and g, using country b as the base. For example, the measured price indexes obtained using the MST method with $1/S_{jk}^P$ as weights differ on average by 20.7 percent from those obtained using $1/\overline{\mathrm{Var}}(\tilde{\alpha}_{jk})$ as weights, when the USA is the base country. The problem with this measure is that the results are not invariant to the choice of base country as can be seen in Table 3. Indeed L_{fg}^b ranges between 18.9 percent (when Japan is the base) and 96.1 percent (when Guinea is the base). An overall measure of dissimilarity (L_{fg}) is obtained by averaging the results for L_{fg}^b across all possible base countries:

$$L_{fg} = \frac{1}{K} \sum_{b=1}^{K} L_{fg}^b.$$

The results for L_{fg}^b and L_{fg} are shown in Table 3. From Table 3 it is clear that the MST price indexes are more sensitive to the choice of weights than are the weighted-EKS price indexes. On average, the MST price indexes change by 29.1 percent, depending on whether $1/S^P_{jk}$ or $1/\overline{\text{Var}}(\tilde{\alpha}_{jk})$ is used as weights, while the weighted-EKS price indexes change by only 2.1 percent. Even a 2.1 percent average change, however, is quite significant. Hence for data sets containing lots of gaps, the choice between $1/S^P_{jk}$ and $1/\overline{\text{Var}}(\tilde{\alpha}_{jk})$ as measures of reliability can have a major impact on the resulting price indexes.

Insert Table 3 Here

8. Conclusion

The latest round of the ICP is attempting to make detailed comparisons of price levels across almost all countries in the world (see ICP Handbook, 2004, and Diewert, 2004a). Even though this comparison is being broken up into regional blocs, obtaining complete matrices of prices for all the countries in each regional bloc is a major undertaking, which may result in either excessive aggregation of data or loss of characteristicity (i.e., countries may be forced to supply price data on products that are not representative of their consumption patterns). This is a problem that arises frequently in international comparisons. It may be counterproductive to try and eliminate all gaps. We have shown here that gaps (i.e., missing observations) in the data are not an insurmountable problem, particularly when weighted binary-based multilateral methods are used. However, it is important that explicit account is taken of these gaps when deciding how much weight is given to each bilateral comparison in the overall multilateral comparison. Failure to make such an adjustment may compromise weighted binary-based methods precisely when they are most needed (i.e., in a comparison over a heterogeneous set of countries). We have developed a method, with strong theoretical foundations, that automatically makes such an adjustment. Our weights, which are derived from the standard errors on Törnqvist price indexes, naturally penalize bilateral comparisons containing many gaps. In the process of developing our method, we have also forged new links between the stochastic and axiomatic approaches to index numbers.

References

Allen, R. C. and W. E. Diewert (1981), "Direct Versus Implicit Superlative Index Number Formulae," *Review of Economics and Statistics* 63(3), 430-435.

Balk. B. M. (1996), "A Comparison of Ten Methods for Multilateral International Price and Volume Comparison," *Journal of Official Statistics* 12(2), 199-222.

Balk, B. M. (1995), "Axiomatic Price Index Theory: A Survey," *International Statistical Review* 63, 69-93.

Caves, D. W., L. R. Christensen and W. E. Diewert (1982), "Multilateral Comparisons of Output, Input and Productivity using Superlative Index Numbers," *The Economic Journal* 92, 73-86.

Clements, K. W. and H. Y. Izan (1981), "A Note on Estimating Divisia Index Numbers," *International Economic Review* 22, 745-747.

Cuthbert, J. R. and M. Cuthbert (1989), *On Aggregation Methods for Purchasing Power Parities*, Department of Economics and Statistics Working Paper No. 56, OECD, Paris.

Diewert, W. E. (2004a), "On the Stochastic Approach to Linking the Regions in the ICP," Mimeo.

Diewert, W. E. (2004b), "Elementary Indices," Chapter 20 of the Consumer Price Index Manual: Theory and Practice, International Labour Organization, Geneva.

Diewert, W. E. (2002), "Similarity and Dissimilarity Indexes: An Axiomatic Approach," Discussion Paper 02-10, Department of Economics, University of British Columbia.

Diewert, W. E. (2001), "The Consumer Price Index and Index Number Theory: A Survey," Discussion Paper 01-02, Department of Economics, University of British Columbia.

Diewert, W. E. (1999), "Axiomatic and Economic Approaches to International Comparisons," in Heston, A., and R. E. Lipsey (ed.), *International and Interarea Com-*

parisons of Income, Output and Prices. Chicago: NBER, University of Chicago Press, 13-87.

Diewert, W. E. (1995), "On the Stochastic Approach to Index Numbers," Discussion Paper 95-31, Department of Economics, University of British Columbia.

Diewert, W. E. (1976), "Exact and Superlative Index Numbers," *Journal of Econometrics* 4, 115-145.

Eltetö, O. and P. Köves (1964), "On a Problem of Index Number Computation Relating to International Comparison," *Statisztikai Szemle* 42, 507-518.

Geary, R.G. (1958), "A Note on the Comparison of Exchange Rates and PPPs between Countries," *Journal of the Royal Statistical Society*, Series A, 121, 97-99.

Gini, C. (1931), "On the Circular Test of Index Numbers," *International Review of Statistics*, Vol.9, No.2, 3-25.

Hicks, J. R. (1946), *Value and Capital*, Second Edition. Oxford: Clarendon Press.

Hill, R. J. (2004), "Constructing Price Indexes Across Countries and Time: The Case of the European Union," *American Economic Review*, forthcoming.

Hill, R. J. (1999), "Comparing Price Levels Across Countries using Minimum Spanning Trees," *Review of Economics and Statistics* 81(1), 135-142.

Hill, R. J. (1997), "A Taxonomy of Multilateral Methods for Making International Comparisons of Prices and Quantities," *Review of Income and Wealth* 43(1), 49-69.

Iklé, D. M. (1972), "A New Approach to the Index Number Problem," *Quarterly Journal of Economics* 86, 188-211.

International Comparisons Program (2004), Handbook. Washington DC, World Bank, forthcoming.

Khamis, S. H. (1972), "A New System of Index Numbers for National and International Purposes," *Journal of the Royal Statistical Society*, Series A, 135, 96-121.

Leontief, W. (1936) "Composite Commodities and the Problem of Index Numbers," *Econometrica* 4, 39-59.

Rao, D. S. P. (2001), "Weighted EKS and Generalized CPD Methods for Aggregation at Basic Heading Level and Above Basic Heading Level," Paper presented at the Joint World Bank-OECD Seminar on Purchasing Power Parities, Washington DC, 30 January - 2 February.

Rao, D. S. P. (1999), "Weighted EKS Method for Aggregation Below Basic Heading Level Accounting for Representative Commodities," Short Note Prepared for the ICP Unit at the OECD.

Rao, D. S. P. and M. Timmer (2003), "Purchasing Power Parities for Industry Comparisons using Weighted Elteto-Koves-Szulc (EKS) Methods," *Review of Income and Wealth* 49(4), 491-511.

Rao, D. S. P., G. Ypma and B. van Ark (2003), "Agricultural Purchasing Power Parities and International Comparisons of Agricultural Output and Productivity," Mimeo, Groningen Growth and Development Centre.

Selvanathan, E. A. and D. S. P. Rao (1994), *Index Numbers: A Stochastic Approach*. Ann Arbor: University of Michigan Press.

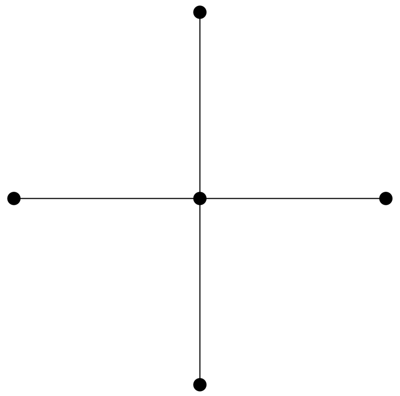
Summers, R. and A. Heston (1991), "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988," *Quarterly Journal of Economics* 106, 327-368.

Szulc, B. (1964), "Indices for Multiregional Comparisons," *Przeglad Statystyczny* 3, *Statistical Review* 3, 239-254.

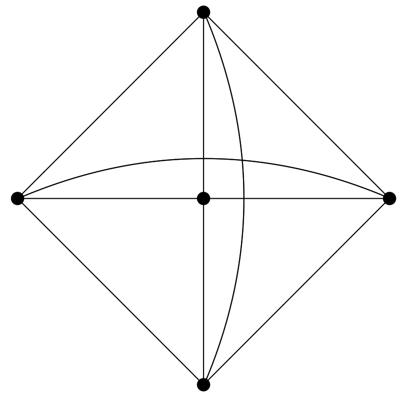
van Veelen, M. (2002), "An Impossibility Theorem Concerning Multilateral International Comparison of Volumes," *Econometrica* 70(1), 369-375.

Wilson, R. J. (1985), *Introduction to Graph Theory*, Third Edition, New York: Longman.

FIGURE 1. — EXAMPLES OF GRAPHS



Star Graph



Complete Graph

FIGURE 2. — EXAMPLES OF SPANNING TREES

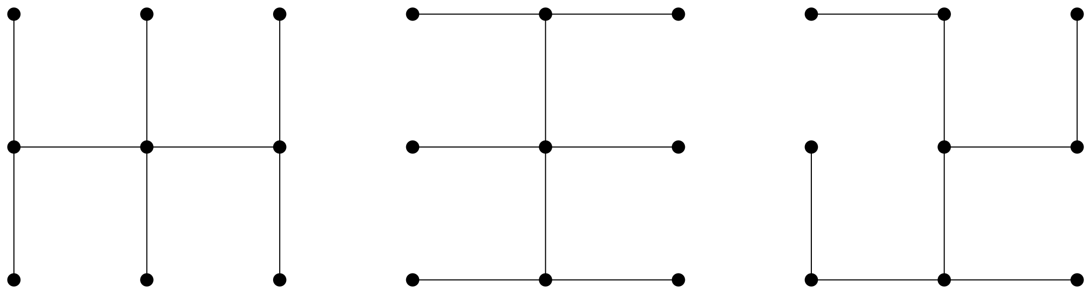


TABLE 1. BILATERAL LINKS IN MINIMUM-SPANNING TREES

MST Bilateral Links		$1/S^P$	N_{jk}	MST Bilateral Links		1/Var	N_{jk}
Norway	Niger	168.96	2	Slovakia	Mexico	208.94	46
Norway	Malaysia	75.16	2	Ireland	Denmark	197.96	25
Sudan	Canada	57.97	5	UK	Ireland	188.76	30
Norway	Albania	52.06	9	Zambia	Tunisia	171.30	18
Norway	New Zealand	48.62	12	Spain	USA	159.91	75
Ireland	Denmark	46.38	25	Norway	Niger	154.54	2
Canada	USA	43.62	24	Senegal	Romania	153.37	19
UK	Ireland	41.58	30	South Africa	Mexico	153.33	57
Finland	Canada	39.26	13	Canada	USA	150.70	24
Peru	Norway	37.15	10	Peru	China	149.58	48
Norway	Indonesia	36.43	3	Spain	Greece	149.53	67
Norway	Costa Rica	34.10	4	Zambia	South Africa	148.92	27
Spain	Norway	30.60	13	South Africa	China	143.76	53
Malaysia	Canada	29.43	4	Spain	Portugal	140.12	55
Switzerland	Malaysia	29.42	13	Morocco	Jordan	139.84	37
Ireland	Germany	29.13	29	Spain	Italy	138.61	66
Norway	Bangladesh	28.70	4	Zambia	Saudi Arabia	133.84	10
Mali	Georgia	27.88	4	Zambia	Jordan	132.88	16
Portugal	Norway	27.85	12	Zambia	Guatemala	132.87	15
Slovenia	Norway	27.81	13	Sweden	Czech Rep	132.85	28
Ghana	Canada	26.89	2	Sweden	Malaysia	131.52	11
Tunisia	Norway	26.43	9	Finland	Canada	130.39	13
Mali	Jordan	26.13	5	Italy	Hungary	129.16	54
Sweden	Canada	25.71	16	Slovakia	Greece	127.77	47
Ukraine	Sweden	25.61	15	Slovakia	Brazil	127.68	32
Saudi Arabia	Norway	25.50	4	UK	Germany	127.29	38
Zambia	Saudi Arabia	23.41	10	Zambia	Switzerland	126.39	13
New Zealand	Mali	22.99	5	Slovenia	Greece	126.26	37
Zambia	Guatemala	22.86	15	Zambia	Nigeria	125.43	24
Ireland	Finland	22.46	24	Sudan	Canada	125.22	5
Ukraine	PNG	22.00	5	Slovakia	El Salvador	120.27	20

TABLE 1. BILATERAL LINKS IN MINIMUM-SPANNING TREES

Poland	Georgia	21.89	11	Slovakia	Czech Rep	120.06	47
Norway	Nepal	20.62	4	UK	USA	118.80	39
Rwanda	Canada	20.17	6	El Salvador	Ecuador	118.44	34
Norway	Myanmar	20.11	4	Greece	Austria	117.16	42
Slovakia	Mexico	19.85	46	Yemen	Slovakia	114.94	27
Canada	Belgium-Lux	19.11	17	Zambia	Chile	114.72	18
Norway	Colombia	18.50	5	Tunisia	Myanmar	108.53	18
New Zealand	Latvia	18.50	11	South Africa	Bolivia	106.27	47
Senegal	Romania	18.34	19	Zambia	Senegal	105.28	22
Norway	Ecuador	18.24	9	Morocco	Egypt	103.69	49
Venezuela	Norway	18.13	6	South Africa	Haiti	101.97	35
Nigeria	New Zealand	17.69	7	Tunisia	Israel	99.87	45
Norway	India	17.47	8	Peru	Norway	99.74	10
Norway	Iran	17.15	7	Venezuela	Guatemala	99.01	19
Norway	El Salvador	17.07	4	Rwanda	China	96.94	24
Norway	Haiti	16.94	4	Mozambique	Morocco	94.59	20
Malaysia	Austria	16.92	15	Slovakia	Panama	94.52	19
Switzerland	Panama	16.86	14	Turkey	Morocco	92.17	52
Slovakia	Malaysia	16.72	15	Norway	Indonesia	92.09	3
Norway	China	16.30	10	Iran	Algeria	91.61	33
Ghana	Burundi	16.23	11	Turkey	Philippines	91.60	33
Slovakia	Brazil	16.05	32	Poland	Mexico	91.29	30
Sweden	Czech Rep	16.01	28	Mali	Jordan	90.90	5
Senegal	Saudi Arabia	15.92	9	Thailand	Slovakia	90.00	24
Switzerland	Mozambique	15.77	8	Tunisia	Iran	89.85	34
Norway	Italy	15.04	13	Czech Rep	Belgium-Lux	89.45	35
Thailand	Myanmar	14.13	24	Peru	India	88.85	43
Zambia	South Africa	14.03	27	Madagascar	Colombia	86.72	29
Switzerland	Congo	13.79	5	Ukraine	Sweden	86.43	15
Norway	Bolivia	13.64	10	Slovakia	Australia	86.24	49
Norway	Greece	13.31	12	New Zealand	China	84.86	20
Mozambique	Chile	13.24	13	Norway	Albania	84.01	9

TABLE 1. BILATERAL LINKS IN MINIMUM-SPANNING TREES

Mozambique	Morocco	12.94	20	Poland	Georgia	83.32	11
Pakistan	Norway	12.78	8	Bulgaria	USA	82.54	54
Slovakia	Hungary	12.56	51	Zambia	Madagascar	77.92	24
Norway	Algeria	12.33	9	UK	France	75.98	39
Saudi Arabia	Madagascar	12.16	10	Tunisia	Bangladesh	75.02	31
Philippines	New Zealand	12.03	11	Latvia	Czech Rep	73.45	24
UK	Netherlands	11.80	28	Senegal	Côte d'Ivoire	72.36	21
Mali	Israel	11.73	7	Uruguay	Nigeria	70.79	21
Norway	Cyprus	11.49	10	Senegal	Pakistan	70.00	22
Norway	Korea	11.29	10	Sudan	Nicaragua	69.93	22
Uruguay	Niger	11.00	14	Ghana	Burundi	66.02	11
UK	France	10.42	39	Portugal	Korea	64.43	35
Yemen	Slovakia	10.22	27	Paraguay	India	64.20	37
Slovakia	Australia	9.78	49	Philippines	Burundi	62.19	20
Guinea	Georgia	9.38	4	Dominican R	Albania	62.01	16
Tanzania	Canada	9.02	9	Tanzania	Czech Rep	58.12	21
Uganda	Saudi Arabia	8.98	8	Norway	Nepal	57.05	4
Thailand	Sri Lanka	8.95	29	Zimbabwe	Burundi	56.79	19
Norway	Dominican R	8.88	4	Greece	Cyprus	55.99	48
Saudi Arabia	Egypt	8.51	11	Saudi Arabia	Costa Rica	51.71	7
Myanmar	Cambodia	8.39	20	Slovakia	Malawi	50.22	18
Russia	New Zealand	8.30	21	Mozambique	Guinea	48.53	11
Canada	Cameroon	8.27	6	Iraq	Egypt	46.38	54
Mali	Japan	7.93	7	Greece	Cambodia	46.01	20
Syria	Mali	7.78	8	Russia	Portugal	44.85	26
Norway	Kenya	7.73	6	PNG	Niger	43.23	12
Sudan	Nicaragua	7.71	22	Uganda	Saudi Arabia	42.68	8
Turkey	Norway	7.68	12	Honduras	Bolivia	40.91	30
Senegal	Côte d'Ivoire	7.56	21	Kenya	Iran	40.28	19
Saudi Arabia	Paraguay	7.46	10	Senegal	Cuba	39.61	19
Zimbabwe	Burundi	7.30	19	Thailand	Sri Lanka	37.54	29
New Zealand	Bulgaria	7.15	22	UK	Netherlands	36.59	28

TABLE 1. BILATERAL LINKS IN MINIMUM-SPANNING TREES

Senegal	Cuba	7.03	19	Switzerland	Congo	34.56	5
Slovakia	Malawi	6.26	18	New Zealand	Japan	33.33	18
Honduras	Ghana	6.08	14	Syria	Algeria	28.98	25
Tanzania	Argentina	3.38	33	Tanzania	Argentina	27.17	33
Canada	Benin	3.35	2	Canada	Cameroon	22.53	6
Iraq	Egypt	2.78	54	Viet Nam	Haiti	7.81	21
Viet Nam	Haiti	1.51	21	Canada	Benin	3.60	2

TABLE 2. PRICE INDEXES (USA=1)

	MST(S^P)	MST(Var)	WEKS(S^P)	WEKS(Var)	EKS	Max/Min
United States of America	1.0	1.0	1.0	1.0	1.0	1.00
Albania	107.1	132.8	153.9	159.7	166.3	1.55
Algeria	65.1	54.1	61.9	59.1	61.2	1.20
Argentina	1.1	0.8	1.0	1.0	1.0	1.39
Australia	1.3	1.3	1.4	1.4	1.4	1.14
Austria	17.8	19.0	20.1	20.1	19.7	1.13
Bangladesh	32.3	40.3	38.8	39.4	37.6	1.25
Belgium-Luxembourg	47.1	41.1	48.3	48.5	48.3	1.18
Benin	288.6	288.6	193.9	186.5	159.2	1.81
Bolivia	3.2	3.4	3.6	3.6	3.5	1.12
Brazil	0.9	0.9	0.9	0.9	0.9	1.08
Bulgaria	7.2	14.1	13.1	13.2	13.4	1.95
Burundi	128.6	144.5	166.6	164.2	156.3	1.30
Cambodia	4208.0	3341.7	3292.8	3429.5	3260.8	1.29
Cameroon	488.5	488.5	2375.8	2209.5	4446.5	9.10
Canada	1.7	1.7	1.9	1.8	1.9	1.12
Chile	371.9	460.7	427.3	430.6	435.2	1.24
China	4.1	4.9	5.0	5.0	4.9	1.22
Colombia	560.4	569.0	680.8	681.2	677.3	1.22
Congo, Dem Republic of	1435.0	1744.6	1931.1	2031.3	3368.7	2.35
Costa Rica	202.3	170.8	215.1	213.0	237.2	1.39
Côte d'Ivoire	495.9	397.0	407.1	403.3	396.7	1.25
Cuba	3.2	2.6	2.6	2.5	6.1	2.47
Cyprus	0.6	0.7	0.7	0.7	0.7	1.26
Czech Republic	26.1	21.7	23.4	23.8	23.1	1.20
Denmark	10.4	10.4	10.1	10.1	9.9	1.06
Dominican Republic	15.3	16.0	19.0	19.0	19.0	1.24
Ecuador	2245.9	2320.0	2308.8	2265.9	2244.3	1.03
Egypt	3.7	2.9	3.4	3.3	3.3	1.30
El Salvador	9.3	10.2	9.8	9.7	9.3	1.10
Finland	11.4	11.4	11.4	11.3	11.2	1.01

TABLE 2. PRICE INDEXES (USA=1)

France	8.8	8.8	9.4	9.5	9.6	1.10
Georgia	3.0	4.7	6.0	6.1	6.5	2.19
Germany	2.2	2.3	2.3	2.3	2.2	1.05
Ghana	247.7	278.3	341.1	337.6	330.7	1.38
Greece	329.3	430.0	462.0	470.1	471.9	1.43
Guatemala	6.2	5.4	5.7	5.7	5.6	1.14
Guinea	580.5	1213.1	1124.4	1155.8	1087.1	2.09
Haiti	11.8	19.2	20.2	20.4	20.0	1.74
Honduras	5.6	6.5	7.3	7.1	7.1	1.29
Hungary	75.6	82.5	87.2	86.5	85.7	1.15
India	17.5	19.7	19.6	19.5	19.3	1.13
Indonesia	2116.8	2624.8	2663.7	2719.5	2623.1	1.28
Iran, Islamic Rep of	1130.7	1138.1	1400.4	1378.9	1449.7	1.28
Iraq	1029.0	793.2	953.7	907.6	941.4	1.30
Ireland	0.9	0.9	0.9	0.9	0.9	1.02
Israel	3.4	4.0	3.8	3.8	3.8	1.17
Italy	1908.4	2360.9	2583.0	2567.8	2570.2	1.35
Japan	501.4	533.5	590.8	577.5	610.5	1.22
Jordan	0.7	0.9	0.9	0.9	0.9	1.25
Kenya	24.2	29.4	33.2	33.4	33.1	1.38
Korea, Republic of	1698.9	2394.7	2652.6	2638.4	2632.6	1.56
Latvia	0.3	0.4	0.4	0.5	0.4	1.51
Madagascar	1549.2	1259.7	1360.9	1348.2	1304.7	1.23
Malawi	5.3	5.6	7.1	7.1	7.2	1.35
Malaysia	3.1	2.9	3.7	3.7	3.6	1.27
Mali	177.0	213.3	307.5	308.4	320.6	1.81
Mexico	4.3	4.6	4.6	4.6	4.4	1.06
Morocco	14.1	13.2	15.4	15.2	15.5	1.17
Mozambique	980.6	920.2	1059.5	1055.0	1029.0	1.15
Myanmar	71.6	77.6	66.9	67.6	64.9	1.19
Nepal	20.6	25.5	21.5	21.1	21.7	1.24
Netherlands	4.0	4.0	3.9	4.1	4.1	1.04

TABLE 2. PRICE INDEXES (USA=1)

New Zealand	1.1	1.6	1.6	1.6	1.6	1.46
Nicaragua	6.2	6.2	9.2	9.2	10.0	1.61
Niger	566.0	701.9	708.3	692.8	692.1	1.25
Nigeria	20.7	42.2	43.8	45.4	43.9	2.20
Norway	13.9	17.2	17.4	17.6	17.5	1.27
Pakistan	14.0	18.8	18.2	18.3	18.6	1.34
Panama	1.6	1.7	1.7	1.7	1.7	1.07
Papua New Guinea	3.2	2.6	3.0	3.0	3.0	1.26
Paraguay	1633.1	1913.1	1682.6	1669.4	1643.0	1.17
Peru	1.8	2.2	2.3	2.3	2.2	1.25
Philippines	22.8	31.1	35.7	35.9	34.9	1.58
Poland	1.1	1.7	2.0	1.9	1.9	1.79
Portugal	185.6	236.2	246.0	247.1	248.7	1.34
Romania	1438.7	1151.9	1206.5	1209.3	1209.2	1.25
Russian Federation	5.4	6.9	7.6	7.5	7.8	1.43
Rwanda	358.5	378.9	398.7	389.8	385.1	1.11
Saudi Arabia	15.2	13.3	15.2	14.9	15.5	1.17
Senegal	556.9	445.9	475.6	471.1	464.2	1.25
Slovakia	23.2	24.5	26.2	26.2	25.9	1.13
Slovenia	106.9	159.0	153.0	155.9	151.9	1.49
South Africa	4.8	4.2	4.3	4.3	4.3	1.14
Spain	140.9	189.9	199.0	201.0	204.2	1.45
Sri Lanka	42.3	44.8	45.7	46.3	46.1	1.10
Sudan	192.6	192.6	227.7	226.0	225.8	1.18
Sweden	10.9	9.1	10.8	10.7	10.7	1.20
Switzerland	3.8	4.6	4.4	4.5	4.5	1.22
Syrian Arab Republic	21.7	25.9	33.6	32.9	33.7	1.55
Tanzania, United Rep of	392.9	282.2	347.7	342.2	338.5	1.39
Thailand	26.1	27.6	26.8	27.1	26.0	1.06
Tunisia	1.1	1.2	1.2	1.2	1.2	1.14
Turkey	46510.3	61390.1	66295.0	66140.0	66221.0	1.43
Uganda	3413.7	2996.0	3148.4	2980.8	2809.5	1.22

TABLE 2. PRICE INDEXES (USA=1)

Ukraine	2.4	2.0	2.4	2.4	2.5	1.22
United Kingdom	1.0	1.0	0.9	0.9	0.9	1.04
Uruguay	4.1	5.4	5.3	5.4	5.4	1.32
Venezuela, Boliv Rep of	192.3	210.1	212.9	212.7	209.1	1.11
Viet Nam	18636.1	30417.0	22198.0	21364.0	16180.0	1.88
Yemen	105.7	111.7	118.3	118.9	117.4	1.13
Zambia	73.0	64.1	63.9	64.4	62.9	1.16
Zimbabwe	2.1	2.4	3.7	3.7	3.7	1.75

TABLE 3. DISSIMILARITY IN THE RESULTS ACROSS MULTILATERAL METHODS

Base Country (b)	L_{fg}^{b} (WEKS)	L_{fg}^{b} (MST)	Base Country (b)	L_{fg}^{b} (WEKS)	L_{fg}^{b} (MST)
USA	1.43	20.67	Germany	1.51	19.00
Albania	3.99	23.50	Ghana	1.70	19.49
Algeria	4.57	35.63	Greece	2.23	27.77
Argentina	2.43	55.35	Guatemala	1.45	30.17
Australia	2.02	18.94	Guinea	3.13	96.05
Austria	1.43	18.92	Haiti	1.65	54.40
Bangladesh	2.08	23.84	Honduras	3.29	20.30
Belgium-Lux	1.50	30.74	Hungary	1.55	19.04
Benin	3.94	20.67	India	1.46	19.55
Bolivia	1.43	18.98	Indonesia	2.52	23.50
Brazil	1.47	18.94	Iran	1.99	20.42
Bulgaria	1.47	82.80	Iraq	4.96	45.03
Burundi	1.93	19.49	Ireland	1.43	21.03
Cambodia	4.38	41.02	Israel	1.67	20.76
Cameroon	7.35	20.67	Italy	1.50	23.36
Canada	3.36	20.67	Japan	2.50	18.91
Chile	1.63	23.44	Jordan	1.48	21.94
China	1.49	21.53	Kenya	1.56	22.35
Colombia	1.43	20.11	Korea	1.48	35.60
Congo	5.40	22.36	Latvia	2.16	35.36
Costa Rica	1.67	33.98	Madagascar	1.64	38.12
Côte d'Ivoire	1.64	39.96	Malawi	1.43	18.94
Cuba	2.69	39.96	Malaysia	1.71	24.03
Cyprus	1.89	24.94	Mali	1.46	21.94
Czech Republ	2.47	35.31	Mexico	1.45	18.94
Denmark	1.43	21.03	Morocco	1.59	24.82
Dominican R	1.45	19.16	Mozambique	1.46	24.82
Ecuador	2.20	19.56	Myanmar	1.71	18.98
Egypt	2.20	45.03	Nepal	2.18	23.50
El Salvador	1.59	19.09	Netherlands	4.25	21.03
Finland	1.50	20.67	New Zealand	2.02	39.97
France	2.32	21.03	Nicaragua	1.50	20.67
Georgia	2.72	49.66	Niger	2.44	23.50

TABLE 3. DISSIMILARITY IN THE RESULTS ACROSS MULTILATERAL METHODS

Nigeria	3.82	91.88
Norway	1.97	23.50
Pakistan	1.47	30.28
Panama	1.43	18.91
PNG	1.73	40.77
Paraguay	1.57	20.75
Peru	1.43	23.50
Philippines	1.56	32.13
Poland	1.50	49.66
Portugal	1.50	25.53
Romania	1.45	39.96
Russia	1.67	25.86
Rwanda	2.49	18.95
Saudi Arabia	2.36	30.17
Senegal	1.64	39.96
Slovakia	1.43	18.94
Slovenia	2.36	41.97
South Africa	1.67	30.17
Spain	1.75	30.77
Sri Lanka	2.02	18.92
Sudan	1.55	20.67
Sweden	1.48	35.31
Switzerland	2.02	22.36
Syria	2.25	21.42
Tanzania	2.00	55.35
Thailand	1.70	18.92
Tunisia	1.46	19.89
Turkey	1.44	28.76
Uganda	5.48	30.17
Ukraine	2.56	35.31
UK	1.48	21.03
Uruguay	2.74	28.46
Venezuela	1.43	19.05
Viet Nam	3.87	54.40
Yemen	1.52	18.94
Zambia	1.58	30.17
Zimbabwe	1.48	19.49

L_{fg} (WEKS)	L_{fg} (MST)
2.13	29.06

Papers issued in the series of the Groningen Growth and Development Centre

Papers marked * are also available in pdf-format on the internet: http://www.ggdc.net/

536 (GD-1)	Maddison, Angus and Harry van Ooststroom, The International Comparison of Value Added, Productivity and Purchasing Power Parities in Agriculture (1993)
537 (GD-2)	Mulder, Nanno and Angus Maddison, The International Comparison of Performance in Distribution: Value Added, Labour Productivity and PPPs in Mexican and US Wholesale and Retail Trade 1975/7 (1993)
538 (GD-3)*	Szirmai, Adam, Comparative Performance in Indonesian Manufacturing, 1975-90 (1993)
549 (GD-4)	de Jong, Herman J., Prices, Real Value Added and Productivity in Dutch Manufacturing, 1921-1960 (1993)
550 (GD-5)	Beintema, Nienke and Bart van Ark, Comparative Productivity in East and West German Manufacturing before Reunification (1993)
567 (GD-6)*	Maddison, Angus and Bart van Ark, The International Comparison of Real Product and Productivity (1994)
568 (GD-7)	de Jong, Gjalt, An International Comparison of Real Output and Labour Productivity in Manufacturing in Ecuador and the United States, 1980 (1994)
569 (GD-8)	van Ark, Bart and Angus Maddison, An International Comparison of Real Output, Purchasing Power and Labour Productivity in Manufacturing Industries: Brazil, Mexico and the USA in 1975 (1994) (second edition)
570 (GD-9)	Maddison, Angus, Standardised Estimates of Fixed Capital Stock: A Six Country Comparison (1994)
571 (GD-10)*	van Ark, Bart and Remco D.J. Kouwenhoven, Productivity in French Manufacturing: An International Comparative Perspective (1994)
572 (GD-11)	Gersbach, Hans and Bart van Ark, Micro Foundations for International Productivity Comparisons (1994)
573 (GD-12)*	Albers, Ronald, Adrian Clemens and Peter Groote, Can Growth Theory Contribute to Our Understanding of Nineteenth Century Economic Dynamics (1994)
574 (GD-13)*	de Jong, Herman J. and Ronald Albers, Industrial Output and Labour Productivity in the Netherlands, 1913-1929: Some Neglected Issues (1994)
575 (GD-14)	Mulder, Nanno, New Perspectives on Service Output and Productivity: A Comparison of French and US Productivity in Transport, Communications Wholesale and Retail Trade (1994)
576 (GD-15)	Maddison, Angus, Economic Growth and Standards of Living in the Twentieth Century (1994)
577 (GD-16)	Gales, Ben, In Foreign Parts: Free-Standing Companies in the Netherlands around the First World War (1994)
578 (GD-17)	Mulder, Nanno, Output and Productivity in Brazilian Distribution: A Comparative View (1994)
579 (GD-18)	Mulder, Nanno, Transport and Communication in Mexico and the United States: Value Added, Purchasing Power Parities and Productivity (1994)
580 (GD-19)	Mulder, Nanno, Transport and Communications Output and Productivity in Brazil and the USA, 1950-1990 (1995)

581 (GD-20) Szirmai, Adam and Ren Ruoen, China's Manufacturing Performance in Comparative Perspective, 1980-1992 (1995)

GD-21 Fremdling, Rainer, Anglo-German Rivalry on Coal Markets in France, the Netherlands and Germany, 1850-1913 (December 1995)

GD-22* Tassenaar, Vincent, Regional Differences in Standard of Living in the Netherlands, 1800-1875. A Study Based on Anthropometric Data (December 1995)

GD-23 van Ark, Bart, Sectoral Growth Accounting and Structural Change in Postwar Europe (December 1995)

GD-24* Groote, Peter, Jan Jacobs and Jan Egbert Sturm, Output Responses to Infrastructure in the Netherlands, 1850-1913 (December 1995)

GD-25* Groote, Peter, Ronald Albers and Herman de Jong, A Standardised Time Series of the Stock of Fixed Capital in the Netherlands, 1900-1995 (May 1996)

GD-26* van Ark, Bart and Herman de Jong, Accounting for Economic Growth in the Netherlands since 1913 (May 1996)

GD-27* Maddison, Angus and D.S. Prasada Rao, A Generalized Approach to International Comparisons of Agricultural Output and Productivity (May 1996)

GD-28 van Ark, Bart, Issues in Measurement and International Comparison of Productivity - An Overview (May 1996)

GD-29* Kouwenhoven, Remco, A Comparison of Soviet and US Industrial Performance, 1928-90 (May 1996)

GD-30 Fremdling, Rainer, Industrial Revolution and Scientific and Technological Progress (December 1996)

GD-31 Timmer, Marcel, On the Reliability of Unit Value Ratios in International Comparisons (December 1996)

GD-32 de Jong, Gjalt, Canada's Post-War Manufacturing Performance: A Comparison with the United States (December 1996)

GD-33 Lindlar, Ludger, "1968" and the German Economy (January 1997)

GD-34 Albers, Ronald, Human Capital and Economic Growth: Operationalising Growth Theory, with Special Reference to The Netherlands in the 19th Century (June 1997)

GD-35* Brinkman, Henk-Jan, J.W. Drukker and Brigitte Slot, GDP per Capita and the Biological Standard of Living in Contemporary Developing Countries (June 1997)

GD-36 de Jong, Herman, and Antoon Soete, Comparative Productivity and Structural Change in Belgian and Dutch Manufacturing, 1937-1987 (June 1997)

GD-37 Timmer, M.P., and A. Szirmai, Growth and Divergence in Manufacturing Performance in South and East Asia (June 1997)

GD-38* van Ark, B., and J. de Haan, The Delta-Model Revisited: Recent Trends in the Structural Performance of the Dutch Economy (December 1997)

GD-39* van der Eng, P., Economics Benefits from Colonial Assets: The Case of the Netherlands and Indonesia, 1870-1958 (June 1998)

GD-40* Timmer, Marcel P., Catch Up Patterns in Newly Industrializing Countries. An International Comparison of Manufacturing Productivity in Taiwan, 1961-1993 (July 1998)

GD-41* van Ark, Bart, Economic Growth and Labour Productivity in Europe: Half a Century of East-West Comparisons (October 1999)

GD-42* Smits, Jan Pieter, Herman de Jong and Bart van Ark, Three Phases of Dutch Economic Growth and Technological Change, 1815-1997 (October 1999)

GD-43* Fremdling, Rainer, Historical Precedents of Global Markets (October 1999)

GD-44* van Ark, Bart, Lourens Broersma and Gjalt de Jong, Innovation in Services. Overview of Data Sources and Analytical Structures (October 1999)

GD-45* Broersma, Lourens and Robert McGuckin, The Impact of Computers on Productivity in the Trade Sector: Explorations with Dutch Microdata (October 1999, Revised version June 2000)

GD-46* Sleifer, Jaap, Separated Unity: The East and West German Industrial Sector in 1936 (November 1999)

GD-47* Rao, D.S. Prasada and Marcel Timmer, Multilateralisation of Manufacturing Sector Comparisons: Issues, Methods and Empirical Results (July 2000)

GD-48* Vikström, Peter, Long term Patterns in Swedish Growth and Structural Change, 1870-1990 (July 2001)

GD-49* Wu, Harry X., Comparative labour productivity performance in Chinese manufacturing, 1952-1997: An ICOP PPP Approach (July 2001)

GD-50* Monnikhof, Erik and Bart van Ark, New Estimates of Labour Productivity in the Manufacturing Sectors of Czech Republic, Hungary and Poland, 1996 (January 2002)

GD-51* van Ark, Bart, Robert Inklaar and Marcel Timmer, The Canada-US Manufacturing Gap Revisited: New ICOP Results (January 2002)

GD-52* Mulder, Nanno, Sylvie Montout and Luis Peres Lopes, Brazil and Mexico's Manufacturing Performance in International Perspective, 1970-98 (January 2002)

GD-53* Szirmai, Adam, Francis Yamfwa and Chibwe Lwamba, Zambian Manufacturing Performance in Comparative Perspective (January 2002)

GD-54* Fremdling, Rainer, European Railways 1825-2001, an Overview (August 2002)

GD-55* Fremdling, Rainer, Foreign Trade-Transfer-Adaptation: The British Iron Making Technology on the Continent (Belgium and France) (August 2002)

GD-56* van Ark, Bart, Johanna Melka, Nanno Mulder, Marcel Timmer and Gerard Ypma, ICT Investments and Growth Accounts for the European Union 1980-2000 (September 2002)

GD-57* Sleifer, Jaap, A Benchmark Comparison of East and West German Industrial Labour Productivity in 1954 (October 2002)

GD-58* van Dijk, Michiel, South African Manufacturing Performance in International Perspective, 1970-1999 (November 2002)

GD-59* Szirmai, A., M. Prins and W. Schulte, Tanzanian Manufacturing Performance in Comparative Perspective (November 2002)

GD-60* van Ark, Bart, Robert Inklaar and Robert McGuckin, “Changing Gear” Productivity, ICT and Services: Europe and the United States (December 2002)

GD-61* Los, Bart and Timmer, Marcel, The ‘Appropriate Technology’ Explanation of Productivity Growth Differentials: An Empirical Approach (April 2003)

GD-62* Hill, Robert J., Constructing Price Indexes Across Space and Time: The Case of the European Union (May 2003)

GD-63* Stuivenwold, Edwin and Marcel P. Timmer, Manufacturing Performance in Indonesia, South Korea and Taiwan before and after the Crisis; An International Perspective, 1980-2000 (July 2003)

GD-64* Inklaar, Robert, Harry Wu and Bart van Ark, “Losing Ground”, Japanese Labour Productivity and Unit Labour Cost in Manufacturing in Comparison to the U.S. (July 2003)

GD-65* van Mulligen, Peter-Hein, Alternative Price Indices for Computers in the Netherlands using Scanner Data (July 2003)

GD-66* van Ark, Bart, The Productivity Problem of the Dutch Economy: Implications for Economic and Social Policies and Business Strategy (September 2003)

GD-67* Timmer, Marcel, Gerard Ypma and Bart van Ark, IT in the European Union, Driving Productivity Divergence?

GD-68* Inklaar, Robert, Mary O'Mahony and Marcel P. Timmer, ICT and Europe's Productivity Performance, Industry-level Growth Accounts Comparisons with the United States (December 2003)

GD-69* van Ark, Bart and Marcin Piatkowski, Productivity, Innnovation and ICT in Old and New Europe (March 2004)

GD-70* Dietzenbacher, Erik, Alex Hoen, Bart Los and Jan Meist, International Convergence and Divergence of Material Input Structures: An Industry-level Perspective (April 2004)

GD-71* van Ark, Bart, Ewout Frankema and Hedwig Duteweerd, Productivity and Employment Growth: An Empirical Review of Long and Medium Run Evidence (May 2004)

GD-72* Edquist, Harald, The Swedish ICT Miracle: Myth or Reality? (May 2004)

Groningen Growth and Development Centre Research Monographs

Monographs marked * are also available in pdf-format on the internet

No. 1*	van Ark, Bart, International Comparisons of Output and Productivity: Manufacturing Productivity Performance of Ten Countries from 1950 to 1990 (1993) (http://www.eco.rug.nl/GGDC/pub/Arkbook.shtml)
No. 2	Pilat, Dirk, The Economics of Catch-Up: The Experience of Japan and Korea (1993)
No. 3	Hofman, André, Latin American Economic Development. A Causal Analysis in Historical Perspective (1998)
No. 4	Mulder, Nanno, The Economic Performance of the Service Sector in Brazil, Mexico and the United States (1999)
No. 5*	Smits, Jan-Pieter, Edwin Horlings and Jan Luiten van Zanden, DutchGNP and Its Components, 1800-1913 (2000) (http://www.eco.rug.nl/GGDC/PUB/dutchgnp.pdf)
No. 6	Sleifer, Jaap, Falling Behind, the East German Economy in Comparison with West Germany from 1936 to 2002 (2003)